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Density-noise power fluctuations in vibrated granular media

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The noise power spectra of the fluctuations in density of a vibrated column of granular material are found to be time dependent. Spectral analysis of these noise power fluctuations shows nontrivial frequency dependences. The noise powers at different frequencies are also found to fluctuate in a partially correlated way. In most instances, the slow variations of the noise are strongly correlated over a broad range of frequencies. These results indicate that highly cooperative interactions exist between fluctuators. In contrast, effects of such strongly coupled fluctuators are absent in the one-dimensional parking-lot-model, one of the simplest systems used to provide a model for recent granular compaction experiments.

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Many of the unusual static and dynamic properties of granular systems stem from the partial stability of a large number of particle configurations (states) at steady-state macroscopic densities [1]. Transitions among these various microscopic states are driven by nonthermal excitations (i.e., mechanical vibration) that lead to fluctuations in observable quantities such as the density. Recent granular compaction experiments [2–4] have shown that the density of a vibrated granular assembly exhibits glassy-like relaxation dynamics and a nontrivial fluctuation spectrum. At issue is the nature of the microscopic dynamics that underlie the macroscopic response of a granular material subject to vertical vibration (tapping).

Probing the interior structure of a three-dimensional (3D) granular assembly, however, is severely hampered by the material's opacity, although amenable to expensive imaging techniques [5]. A complimentary approach has been to compare more readily available data with simulations of models for vibrated powders. In many seemingly different treatments [3,6–13], most of which are based on geometrical models of ''parking'' [6,9] or simulating the motion and subsequent relaxation of particles [7,8,10,11], the resulting dynamics have some striking similarities to the experimental results [2,3,14]. Similarities include: The logarithmic time course of the density relaxation, irreversible-reversible curves, and the power spectrum of density fluctuations. More detailed characterization, however, is needed to determine if the models really capture the processes underlying the observed kinetic behavior.

We have previously reported the average properties of the noise power of density fluctuations in a vibrated column of beads [3]. In this paper we examine the time-dependent variations of the noise power and report on non-Gaussian properties, i.e., higher moment correlation functions of the density fluctuations. These properties can be used to distinguish between different kinetic models that exhibit identical power spectra. The power spectrum of a signal is simply the Fourier transform of the two-point autocorrelation function. For a Gaussian process, all higher-order correlations are uniquely expressible in terms of two-point correlations, which thus contain all possible information about the signal [15]. For systems in which the number of independent fluctuating entities is relatively small, the non-Gaussian properties of higher-order correlations contain additional information [16]. The statistical signatures we measure clearly demonstrate that the noise power is sensitive to a small number of effective fluctuators that exhibit cooperative kinetics.

The results presented here are based on time-series of the density fluctuations from previously reported work on granular compaction [3], and so we shall not comment extensively upon the experiment. Briefly, the experiment involved monodisperse spherical glass beads that were confined inside a cylindrical tube that was vibrated vertically with discrete shakes or ''taps'' with an intensity parametrized by $\Gamma = a/g$, where a is the peak acceleration during a tap and g is the gravitational acceleration. The fraction of volume occupied by the beads, or equivalently, the density, was measured after each tap near the top, middle, and bottom of the tube using a capacitance technique that averaged over roughly 6000 beads. Here, we are concerned with the statistical properties of these density fluctuations after the system has relaxed to a steady-state density $\rho_{ss}(\Gamma)$.

Figure 1 shows an example of the complicated time dependence of the noise power of the density fluctuations, where time is measured in taps. Such time records are obtained as follows. A power spectrum, $S(f)$, of the density fluctuations is calculated by squaring the absolute value of the Fourier transform of a 1008-point time series. Frequency, f, has units of inverse taps. To reduce the amount of numbers to be dealt with and the fractional uncertainty associated with sampling a random signal, each spectrum is summed into seven octaves, O_i with $i = 1,2,\ldots,7$. The lowest octave, O_1, is roughly 0.004–0.007 taps^{-1}, consisting of four power spectra points, and the highest octave, O_7, containing 192 power spectra points ranges from 0.19 to 0.38 taps^{-1}. This procedure was repeated, up to 536 times for our longest density-time series, to generate the time series of octave powers shown in Fig. 1. The noise power magnitude exhibits large variations as a function of time, often with strong cor-

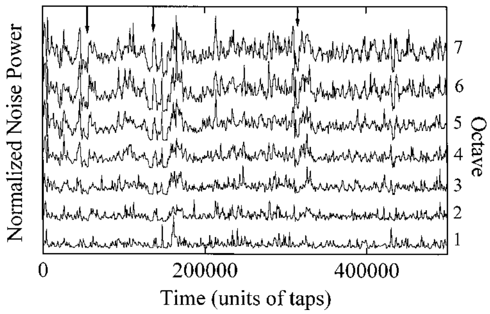

FIG. 1. Noise power as a function of time (measured in taps) is shown for experimental data taken at $\Gamma=5.9$ near the bottom of the column of beads. The noise power for each octave is normalized by the standard deviation of the expected Gaussian variance and offset for clarity. The logarithmic center frequencies (units of taps^{-1}) for each octave are octave 1, 0.0053; 2, 0.0094; 3, 0.018; 4, 0.035; 5, 0.068; 6, 0.14; 7, 0.27.

relations of the noise power between differing octaves (e.g., see features indicated by the arrows in Fig. 1).

The frequency dependence of the fluctuations in noise power can be used to distinguish between systems involving interacting fluctuators and those involving the superposition of a fixed set of independent two-state systems (TSS) [16]. Simple superpositions of single-rate processes with a distribution of rates are often invoked in models for electronic noise [16] and recently for granular compaction [6,9]. Each TSS is fully characterized by two characteristic rates, $1/\tau_1$ and $1/\tau_2$, for switching between the two states and contributes a Lorentzian to the power spectrum, i.e., $S(f)=\tau/(1+4\pi^2 f^2\tau^2)$. Since a TSS has no characteristic frequencies much below $1/\tau=1/\tau_1+1/\tau_2$, fluctuations in noise power in some frequency band near $1/\tau$ will be nearly frequency independent. In contrast, systems with more complicated fluctuators, e.g., interacting TSS having different characteristic switching rates, often exhibit fluctuations in noise power that are frequency dependent.

The frequency components to the noise power fluctuations in Fig. 1 are clearly seen by Fourier transforming and squaring the time record of the noise power for a given octave having a logarithmic center frequency, f_i. For each octave the so called "second-spectrum," $S_2(f_2,f_i)$, is calculated at the frequency f_2, determined by the time scale of the fluctuations in Fig. 1. The excess second-spectrum of the density fluctuations is calculated by subtracting the expected Gaussian contribution (see discussion below and Ref. [17]) from $S_2(f_2,f_i)$. Since each data point making up an octave time series requires 1008 density samplings, practical considerations limit the frequency span of the second-spectrum to 64 spectral points. For clarity, we reduce the statistical fluctuations in amplitude at the expense of frequency resolution by summing the excess $S_2(f_2,f_i)$ into octaves. The resulting spectra for the highest octave, f_7, are plotted in Fig. 2 for the two intensities, $\Gamma=5.9$ and 6.8, which had the longest octave time series. (For the lowest four octaves, the Gaussian background is comparable to the magnitude of $S_2(f_2)$, rendering them unusable.) In this representation, a

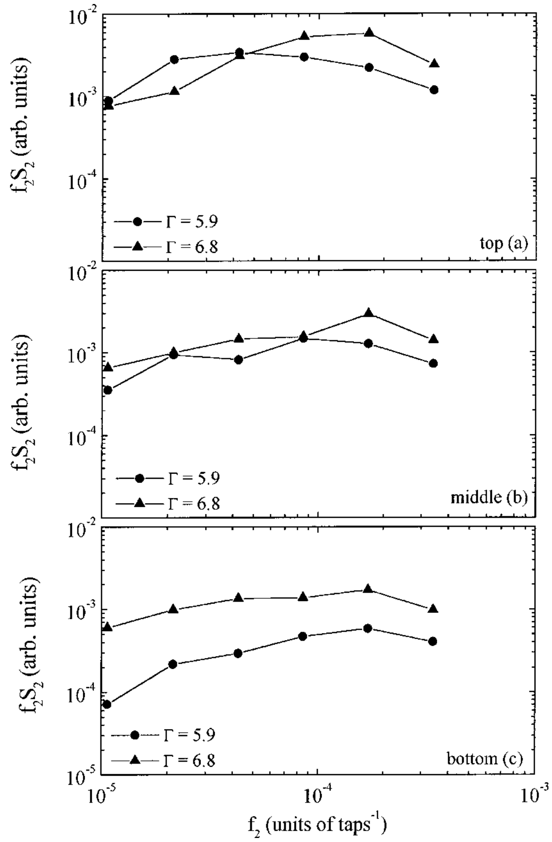

FIG. 2. Non-Gaussian (excess) second spectrum, f_2S_2, of the fluctuations in the noise power in the frequency range 0.19 to 0.38 taps^{-1} (octave 7) for experimental data acquired near (a) the top, (b) the middle, and (c) the bottom of the column of beads. The symbols represent second-spectrum octave sums, so that a $S_2\sim 1/f_2$ dependence appears horizontal on these plots. Similar dependences of S_2 on f_2 are found for octaves 4 through 6. Second-spectra for the lower frequency octaves taken at $\Gamma=5.9$ and 6.8, and for all octaves at $\Gamma=4.3$ and 5.1, were dominated by the expected Gaussian contribution due to the finite number of samples.

$1/f$ spectral dependence will appear flat and a frequency-independent (white) spectrum will have slope $+1$.

The dependence of S_2 on f_2 turns out to be nontrivial at all three depths in the pile. Approximating $S_2(f_2)$ by a power-law $S_2\propto f_2^{-\beta}$, the exponent β ranges from 0.5 to 0.8 near the bottom and middle of the pile. A power-law form for $S_2(f_2)$ is less appropriate at the top of the pile where $S_2(f_2)$ apparently exhibits a broad maximum, reflective of a characteristic rate at which the ordinary power spectrum is being modulated. These second spectra rule out models based on simple superpositions of stationary independent TSS, which would exhibit a white second spectrum [16].

We now consider whether the individual components of $S(f)$, the components that are being modulated on longer timescales, come from two-state systems that contribute Lorentzians to $S(f)$ or from multistate fluctuators, which contribute broader spectra. The correlations in the variations in the noise power between differing frequency octaves provide a direct way of investigating this question. These correlations can be quantified by calculating the covariance matrix. Adapting the analysis of Restle *et al.* [18], we define the elements of the matrix indexed by octaves i and j as

$$CM_{ii}=\langle(\delta O_i)^2\rangle \Big/ \sum_k \langle P_k\rangle^2,$$

and for $i\neq j$

$$CM_{ij}=\langle(\delta O_i)(\delta O_j)\rangle/[\langle(\delta O_i)^2\rangle\langle(\delta O_j)^2\rangle]^{1/2},$$

where O_i is the noise power in octave i, P_k is the power in one of the Fourier frequency bins summed in octave i, $\delta O_i=(O_i-\langle O_i\rangle)$, and $\langle\ \rangle$ indicate the average over the number of spectral sweeps. The diagonal elements are the variances, normalized to give unity for Gaussian noise. The off-diagonal elements are the correlation coefficients for fluctuations between octaves. The values can range from $+1$ to -1, corresponding to exactly positively and negatively correlated noise power, and should be zero for Gaussian noise.

We find significant excess variance (i.e., $CM_{ii}>1$) for the four vibration intensities ($\Gamma=4.3$, 5.1, 5.9, and 6.8) that were previously studied [3]. The variances increase monotonically with octave number and typical values range from 1.5 to 10, which is much larger than the sampling error due to finite number of samples. The excess fractional variance [17], $(\langle\delta O_i^2\rangle-\Sigma_k\langle P_k\rangle^2)/\langle O_i\rangle^2$, is also large ($\sim$0.1), consistent with a small number of fluctuating entities producing most of the noise.

If the diagonal terms show variances well in excess of the Gaussian value, then the correlation coefficients may be rescaled to include only the non-Gaussian component of the variance by $C_{ij}=CM_{ij}(CM_{ii}CM_{jj})^{1/2}/[(CM_{ii}-1)(CM_{jj}-1)]^{1/2}$. In Fig. 3(a) the C_{ij}'s with the same octave separation were averaged and are plotted as a function of octave separation for the various Γ. These show a positive correlation of spectral power fluctuations between different octaves, with the magnitude of the correlation falling off as a function of octave separation.

For a continuous distribution of Lorentzian functions (i.e., TSS) that yield a $1/f$-like frequency dependence for $S(f)$ and for which the amplitude of each Lorentzian is independently and randomly modulated, it has been shown [19] that $C_{ij}=r/\sinh(r)$, where $r=\ln(f_i/f_j)$. This curve is plotted as a thick solid line in Fig. 3 and its functional form is due to the self-convolution of individual Lorentzian power spectra. Figure 3(a) shows that the correlations at large octave separation are considerably stronger than $r/\sinh(r)$ for data taken near the bottom of the pile. At $\Gamma=5.9$, even octave 1 and octave 7 have a correlation coefficient of 0.68, which is more than a factor of 5 larger than independently modulated Lorentzians. These results indicate that the individual components that contribute to the non-Gaussian fluctuations have broader spectra, like those of multistate fluctuators having a range of characteristic frequencies.

For $\Gamma=5.9$ and 6.8, the two intensities for which we have the best statistics, the interoctave correlations were systematically largest near the bottom and top of the pile. However, Fig. 3(a) also shows that $C_{ij}(r)$ falls off more quickly than $r/\sinh(r)$ near the middle of the pile for $\Gamma=5.9$. Such effects

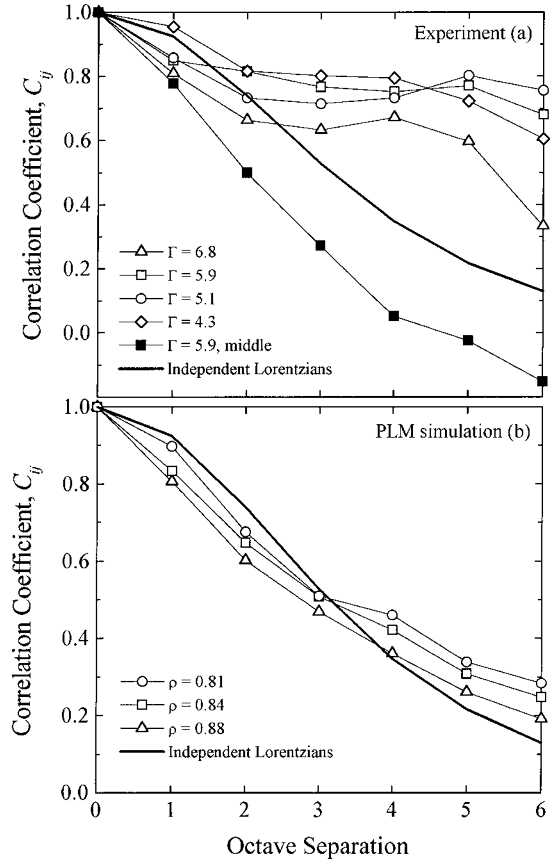

FIG. 3. Shown are the averaged correlation coefficients between non-Gaussian fluctuations of the power in different octaves of the ordinary power spectrum. For comparison, the thick solid line in both panels shows the expected correlations for a superposition of Lorentzians with independently modulated intensities. The experimental data (a) shows the correlations found near the bottom of the column of beads for various shaking intensities. Panel (b) shows the correlations exhibited by simulations of a 1D parking-lot-model (see text) at various steady-state densities, ρ.

can also arise for multistate fluctuators [20] because each particular state partakes to different degrees in the different relaxation eigenmodes. Thus, the spectral weight due to the different relaxation modes itself fluctuates. As a result, there can be a negative contribution to the cross-correlation between frequencies near different relaxation rates. The central point is that the $C_{ij}(r)$ curves we observe indicate that the noise power fluctuations are not due to independent modulations of the amplitudes of a collection of Lorentzians.

Next, we compare the experimental results to the statistical signatures of density noise in simulations of a one-dimensional (1D) version of the parking-lot-model (PLM). These simulations have reproduced many of the essential features found in the experiment. Details of the simulations can be found in Refs. [3,9,21]. Briefly, the desorption process (parking spot opens up) in the PLM simulations is unrestricted, whereas adsorption (a parking event) is subject to free volume constraints, i.e., cars cannot overlap. For high average densities, a net gain of one car requires a cooperative rearrangement of many particles in order to form a space large enough for another car to park. Despite the involvement of many particles, in practice the density fluctuations can be modeled as a superposition of independent fluctuators [9].

When the parking lot is relatively empty, only Gaussian noise is observed indicating that the number of independent fluctuating entities is large. Non-Gaussian noise sets in for $\rho_{ss}>0.8$, for which $CM_{ii}>1$ and the off-diagonal elements are nonzero and positive. This is to be expected since, in the dense limit, the number of events that lead to a net change in density is small. (The experiment is most closely associated with the dense limit.) For steady-state densities ranging from 0.37 to 0.88, the PLM exhibits a white excess second-spectrum. This suggests that the PLM can be described by a superposition of fixed TSS, in sharp contrast to the experiment. Figure 3(b) shows that the cross-correlation coefficients for PLM fall off quickly with octave separation. This behavior is also consistent with fixed, independent TSS [22]; the fact that $C_{ij}(r)$ for the PLM is well described by $r/\sinh(r)$, particularly at the highest density, $\rho_{ss}=0.88$, may be fortuitous. Hence, the noise power fluctuation in the PLM can be thought of as simply coming from independent pulse trains of different characteristics rates.

The discrepancies between the experiment and the 1D PLM simulations probably indicate that in three dimensions any local metastability, associated with some free volume, typically supports more than two metastable states. Furthermore, the high connectivity of 3D leads to coupling between different metastable clusters, perhaps ones with significant spatial separation. Preliminary studies of correlations between particle rearrangements in different regions of the pile [23] show their spatial extent to extend up to the dimension of the system. It is perhaps not surprising that a 1D model completely fails to capture the cooperative nature of fluctuations, considering the failure of 1D models to predict long-range order in phase transitions with finite-range interactions.

In conclusion, the use of the covariance matrix and second-spectral techniques has revealed differences in kinetic behavior between experimental data on granular compaction and a 1D version of the parking-lot-model often used to describe it. The main difference is that the density fluctuations in the PLM can be regarded as a superposition of independent two-state systems, whereas the experimental data cannot be decomposed into two-state systems on any time scale. In the experiment, the nontrivial correlations of the noise power as a function of octave separation are consistent with multistate behavior with coupling between events on different time scales. Further studies of the second spectra on even longer experimental density-time series and in simulations of a 3D assembly of shaken hard spheres [11] will help elucidate the nature of the cooperative particle dynamics and improve our understanding of granular dynamics. The analysis methods presented here should also prove valuable for discriminating among different models for granular compaction.

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